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CHIRAL PARTNERS IN A CHIRALLY BROKEN WORLD

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The isovector–vector and the isovector–axial–vector current are related by a chiral transformation. These currents can be called chiral partners at the fundamental level. In a world where chiral symmetry was not broken, the corresponding current–current correlators would show the same spectral information. In the real world chiral symmetry is spontaneously broken. A prominent peak — the ρ -meson — shows up in the vector spectrum (measured in e^+e^- -collisions and τ -decays). On the other hand, in the axial–vector spectrum a broad bump appears — the a_1 -meson (also accessible in τ -decays). It is tempting to call ρ and a_1 chiral partners at the hadronic level. Strong indications are brought forward that these “chiral partners” do not only differ in mass but even in their nature: The ρ -meson appears dominantly as a quark–antiquark state with small modifications from an attractive pion–pion interaction. The a_1 -meson, on the other hand, can be understood as a meson–molecule state mainly formed by the attractive interaction between pion and ρ -meson. A key issue here is that the meson–meson interactions are fixed by chiral symmetry breaking. It is demonstrated that one can understand the vector and the axial–vector spectrum very well within this interpretation. It is also shown that the opposite cases, namely ρ as a pion–pion molecule or a_1 as a quark–antiquark state lead to less satisfying results. Finally speculations on possible in-medium changes of hadron properties are presented.

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1. Chiral Symmetry Breaking

According to our present knowledge of the strong interaction, chiral symmetry is an approximate symmetry of the interaction but not of the ground state (vacuum). In other words, in the light quark sector chiral symmetry is spontaneously broken (on top of the small explicit breaking due to the non-vanishing quark masses). A strong indication that chiral symmetry is spontaneously broken comes from the study of the isovector–vector current $\vec{j}_V^\mu = \bar{q}\vec{\tau}\gamma^\mu q$ and the isovector–axial–vector current $\vec{j}_A^\mu = \bar{q}\vec{\tau}\gamma_5\gamma^\mu q$. These currents are intertwined by a chiral transformation

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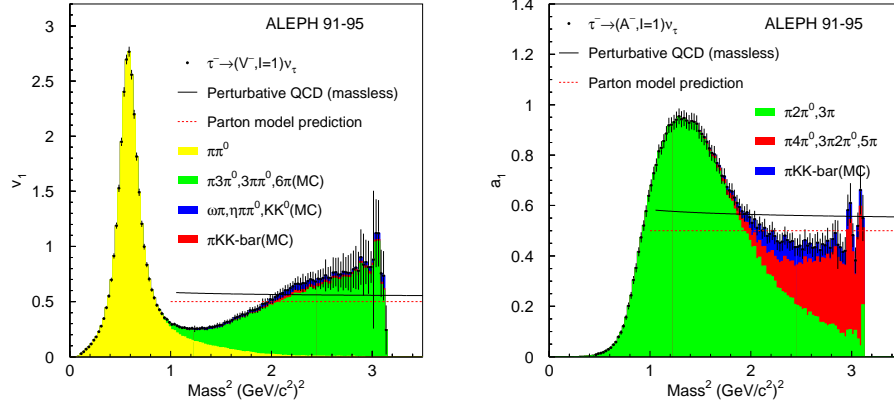


Fig. 1. Spectral information of the vector (*left*) and axial-vector (*right*) current. Figures taken from Ref. 4.

(see e.g. Ref. 1 and references therein). It is illuminating to work out the quantum numbers of these currents:² \vec{j}_V^μ has the quantum numbers $I^G(J^{PC}) = 1^+(1^{--})$, while \vec{j}_A^μ has the quantum numbers $I^G(J^{PC}) = 1^-(1^{++})$.^a These are the quantum numbers of the ρ - and a_1 -meson, respectively.³ Suppose for a moment that chiral symmetry was not broken, i.e. realized in the same way as e.g. isospin symmetry (Wigner-Weyl mode). In this case one would expect (nearly) identical spectra from the current-current correlators of \vec{j}_V and \vec{j}_A . Of course, this statement is not limited to these two currents, but applies also to other quark currents connected by chiral transformations. However, the particular currents \vec{j}_V and \vec{j}_A are exceptional for the following reason: They are directly accessible by electroweak processes. Therefore, experiment can tell us whether the ground state of the strong interaction is in the Wigner-Weyl mode of chiral symmetry. For example, photons couple to the neutral currents contained in \vec{j}_V whereas the hadronic weak current is formed by $\vec{j}_V - \vec{j}_A$. The Fourier decomposition, i.e. the spectral information of the current-current correlators of \vec{j}_V and \vec{j}_A is depicted in Fig. 1. Obviously the spectra are not identical, not even approximately. This is one of the clearest indications that chiral symmetry is spontaneously broken.

2. Chiral Partners at the Level of Hadrons — ρ and a_1 ?

Since the currents \vec{j}_V and \vec{j}_A are connected to each other by chiral transformations, it is suggestive to call them chiral partners at the fundamental level of quark currents. But which objects should one call chiral partners at the level of hadrons? By

^aStrictly speaking the quantum number of charge conjugation C can only be assigned to the neutral currents.

inspecting Fig. 1 we find that the ρ -meson shows up as a prominent peak in the vector spectrum (left panel), while the a_1 -meson appears as a broad bump in the axial-vector spectrum. This suggests to call ρ and a_1 chiral partners. On the other hand, these states do not even have the same mass. While it is natural that “partner spectra” are different for a system where the symmetry is spontaneously broken, there are still states which are energetically degenerate. For a given hadron, e.g. a single-particle hadron state $|h\rangle$, the two-particle state $|\pi(0)h\rangle$ is (approximately) degenerate in energy. Here $\pi(0)$ is a soft pion state, i.e. with vanishing momentum. The Goldstone theorem demands that Goldstone bosons are massless and do not interact for vanishing momentum. Hence the two-particle state $|\pi(0)h\rangle$ is degenerate in energy with the single-particle state $|h\rangle$.^b Should one call a_1 or a π - ρ system the chiral partner of the ρ ? One might regard it merely as a matter of definition. On the other hand, we will demonstrate in Sec. 3 that these two possibilities are actually intimately connected. In addition, the question of chiral partners becomes a real issue in a strongly interacting medium where chiral symmetry restoration is expected to take place. There, the spectra which are shown for vacuum in Fig. 1 must become degenerate. If there were still peak or bump structures visible in the spectra at the point of chiral restoration, then it would be suggestive to trace back from which vacuum structures these in-medium structures emerge. While the in-medium structures must show a degeneracy (chiral restoration!), the corresponding vacuum structures do not. But it is then tempting to call the vacuum structures chiral partners as they become degenerate at the point of chiral restoration. We will come back to this point in Sec. 4 below.

3. Different Nature of Hadronic Chiral Partners

We are aiming at an understanding of the respective low-energy part of the spectra depicted in Fig. 1. Both show a resonant structure: In the vector spectrum (left panel) there is a peak at about 770 MeV called the ρ -meson. In the axial-vector spectrum (right panel) there is a broad bump at about 1250 MeV called the a_1 -meson. Actually both low-energy parts are governed by a (quasi-)two-particle final state — $\pi\pi$ for the vector and $\rho\pi$ for the axial-vector channel. The latter can be deduced from a Dalitz plot analysis of the three-pion final state.⁵ The general strategy is the following: The two-particle state is subject to final-state interactions (rescattering). We want to figure out whether this final-state interaction is sufficient to create the respective resonant structure seen in Fig. 1 or whether one needs in addition a preformed resonance, i.e. an elementary hadronic state which microscopically should be regarded as a quark-antiquark state. This intrinsic structure is, however, not resolved at the hadronic level. We study two scenarios: For the first scenario we only include the final-state interactions which we describe via a Bethe-Salpeter

^bThis degeneracy is broken to some extent due to the explicit symmetry breaking by the finite current quark masses.

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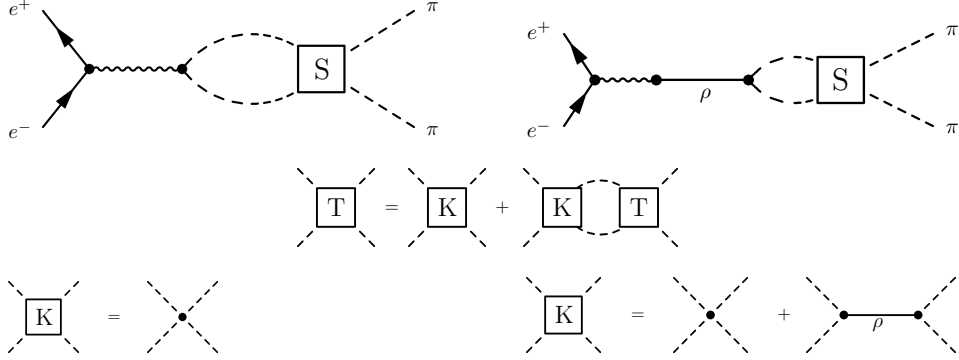


Fig. 2. Description of the electromagnetic form factor of the pion within the two scenarios. The processes from which the form factor is extracted are depicted in the top panels. For the first scenario (only final-state rescattering) only the top left diagram enters. For the second scenario both diagrams in the top line are considered. The boxes labeled with S denote the S -matrix of pion-pion scattering. It is obtained from the T -matrix which in turn results from the solution of a Bethe-Salpeter equation (middle panel). The kernel K of the Bethe-Salpeter equation for the first/second scenario is shown in the bottom left/right panel. In the first scenario this kernel is fixed by the lowest order chiral interaction. It is a point interaction as depicted in the lower panels. In addition, for the second scenario the preformed resonance appears in the kernel.

equation. The kernel is taken from the lowest order chiral interaction. It is important to note that the strength of this final-state interaction is fixed by chiral symmetry breaking and is therefore parameter free. In the second scenario we include in addition a preformed resonance. If we got a reasonable description of the data from the first scenario, we would conclude that the resonance in the considered channel is a dynamically generated state, a meson-meson molecule. Otherwise, we would conclude that the resonance has a non-negligible or even dominant quark-antiquark contribution which can be quantified in the second scenario.

Nature of resonances — the ρ -meson: Instead of the two-pion spectrum of Fig. 1, left, we study the electromagnetic form factor of the pion in the time like region.⁶ Due to isospin symmetry this contains the same information.⁴ The relevant processes are depicted in Fig. 2. The lowest-order chiral interaction of the two-pion system is given by the non-linear sigma model.⁷ We have two parameters at our disposal: the renormalization points (a) for the loop of the transition from the virtual photon to pions (Fig. 2, top left) and (b) for the loop appearing in the Bethe-Salpeter equation (Fig. 2, middle). These renormalization points are not (completely) free, however: First of all, both have to be in a reasonable range (see below). Second, the renormalization point for the loop in the Bethe-Salpeter equation can be fixed by the requirement of approximate crossing symmetry⁸ (see also Ref. 9 for a different line of reasoning which yields the same result). The result of the first scenario (only final-state interaction) is shown in Fig. 3, left. The full line labeled “low μ ” is obtained for both renormalization points set to the pion mass. Obviously, one fails to describe the data. If the renormalization points are

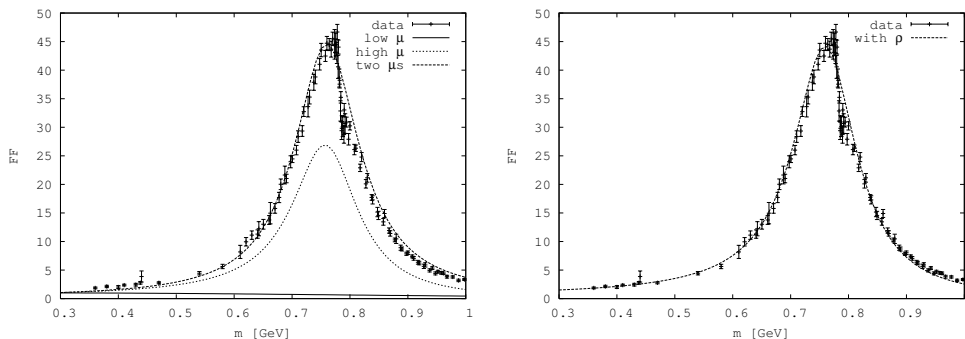


Fig. 3. *Left:* The pion form factor in the first scenario (only rescattering of pions). The physically reasonable calculation is shown by the full line. The other calculations are technically possible, but physically unreasonable, since they correspond to renormalization points in the TeV range. See main text for details. *Right:* The pion form factor in the second scenario where an elementary resonance is included in addition.¹¹ Data taken from Ref. 6.

increased, it is possible to create a peak structure. The dotted line labeled “high μ ” is obtained for both renormalization points set to 1.1 TeV. Finally, one gets the dashed line labeled “two μ s” by setting the first renormalization point (photon-to-pion transition) to 10 TeV and the second one to 1.1 TeV. Thus, from a purely technical point of view the approach allows for a description of the data (dashed line). From the physical point of view, however, it must be stressed that only the full line corresponds to a reasonable calculation, since the renormalization points should lie in a reasonable range and not orders of magnitude away from typical hadronic scales. We conclude that with a physically reasonable choice of parameters one cannot explain the pion form factor within a scenario which includes only pion-pion rescattering. One needs in addition an elementary resonance as we will show next. A similar conclusion has been drawn in Ref. 10 studying the pion-pion scattering phase shifts. We now turn to the second scenario where an elementary resonance is included in addition to the pure rescattering studied in the first scenario. The form factor is now obtained from the sum of diagrams shown in the top line of Fig. 2. The Bethe-Salpeter equation is formally unchanged, Fig. 2, middle, but the kernel is now given by the sum of the point interaction obtained from the non-linear sigma model and the elementary resonance, cf. Fig. 2, bottom left. As additional parameters one has now the mass of the elementary resonance and its couplings to the photon and to two pions. Actually, changes in the renormalization points can be compensated by changes in these resonance parameters. Therefore, one has effectively three free parameters. As shown in Fig. 3, right, one gets an excellent description of the pion form factor.¹¹ In particular, there is no two-peak structure in the theory curve since the pion contact interaction alone is not very strong, as already shown by the full line in Fig. 3, left. We conclude that the ρ -meson is dominantly a preformed (i.e. quark-antiquark) state with a small two-pion admixture and not a pion-pion molecule.

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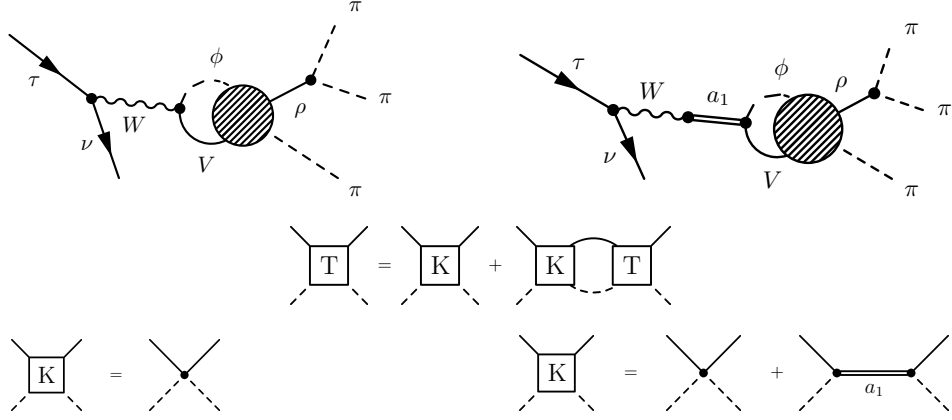


Fig. 4. Diagrams relevant for the description of the process $\tau \rightarrow \nu_\tau + 3\pi$. For details see Fig. 2, main text and Ref. 5.

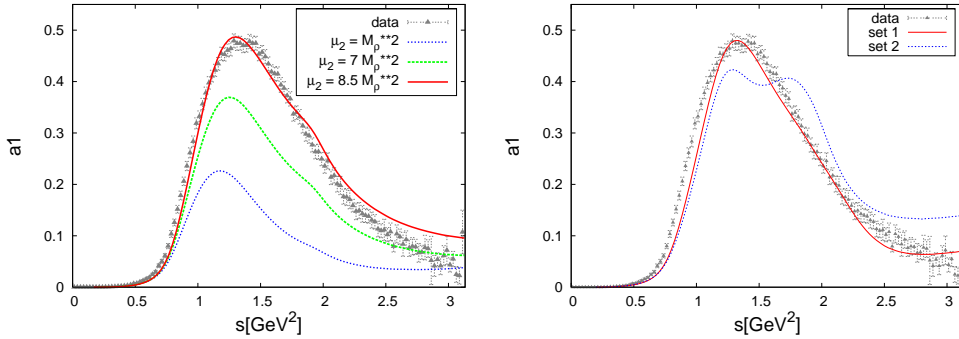


Fig. 5. The axial-vector spectral information in the three-pion final state described by the first (*left*) and the second (*right*) scenario. See main text and Ref. 5 for details. Data taken from Ref. 4.

Nature of resonances — the a_1 -meson: The analysis of the a_1 -meson exactly resembles the one presented for the ρ -meson, but the result will be just the opposite: We will show in the following that the a_1 -meson can be understood as a π - ρ molecule. The relevant processes for the description of the decay $\tau \rightarrow \nu_\tau + 3\pi$ are schematically depicted in Fig. 4. The lowest-order chiral interaction of the ρ - π system is given by the Weinberg-Tomozawa interaction.¹² Results of the calculations for both scenarios are compared to data in Fig. 5. In the first scenario the a_1 -meson emerges as a dynamically generated state from final-state interactions of vector and pseudoscalar mesons. That axial-vector mesons can be created in this way has been suggested in Ref. 8 and later in Ref. 13. In these works a coupled-channel treatment of π - ρ and K - K^* has been presented. We follow this approach, but note in passing that the strangeness channel is not very important for the a_1 -meson.⁵ For the first scenario we take the parameter-free scattering amplitude from Ref. 8. Then we are

left with only one free parameter, the renormalization point μ_2 of the entrance loop from the W -boson to hadrons (cf. Fig. 4, top left). We recall that this parameter should be in a reasonable range. By only tuning μ_2 we get a decent description of the data as shown in Fig. 5, left. This means that we can essentially describe the position, height and width of the a_1 -bump with one parameter which is in the GeV range (and not in the TeV range as for the case of the ρ -meson). In the second scenario where we include in addition an elementary resonance we typically generate a double peak structure (dotted line in Fig. 5, right). This is not surprising since we know from the first scenario that the final-state interaction between ρ and π is strong enough to create a resonance dynamically. An additional elementary resonance can only be hidden, if its coupling to the ρ - π system is weak (which essentially brings back the first scenario) or if its mass is fine-tuned such that it appears at the position of the dynamically generated resonance. The latter possibility is shown as the full line in Fig. 5, right. While this is technically possible we regard it as rather unnatural that a quark-antiquark and a meson-meson state appear at the very same position. Therefore, the natural explanation of the τ -decay data shown in Fig. 5 is that the a_1 -meson is a dynamically generated state, i.e. a meson-meson molecule⁵ as suggested in Refs. 8, 13. To summarize the present section: There are strong indications that the ρ -meson and the a_1 -meson, the “chiral partners” at the level of hadrons, are not only different in mass, but actually different in nature: The ρ -meson is dominantly a quark-antiquark state whereas the a_1 -meson is dominantly a meson-meson state (mostly ρ - π).

4. Outlook to Chiral Restoration

As already discussed in Sec. 2 we expect that the spectral information of the vector and the axial-vector current become identical at the point of chiral restoration. There are various scenarios conceivable how this degenerate in-medium spectrum might look like. Here we briefly discuss only two. The degeneracy scenario: We have seen above that the ρ -meson is dominantly a single-particle state at the hadronic level (and not a pion-pion correlation). If the ρ -meson was still dominantly a single-particle state at the point of chiral restoration (i.e. if it still showed up as a prominent peak in the spectrum), this would require the existence of another single-particle state at the hadronic level with opposite parity, i.e. an axial-vector state. Since we have shown that the a_1 -meson is well described as a two-particle state, a ρ - π correlation, there should be another, i.e. higher-lying axial-vector state which becomes degenerate with the ρ -meson. We cannot exclude this possibility and within our formalism we have not much to say about this scenario. However, there is at least one alternative. The melting scenario: It might appear that the ρ -meson dissolves already in hadronic matter which can be interpreted as a precursor to deconfinement. Then also the a_1 -meson should dissolve. In principle, this can be tested in our approach. In the following we present a very simple model: We increase the width of the ρ -meson by a constant (by 50 or 100 MeV, respectively) and study

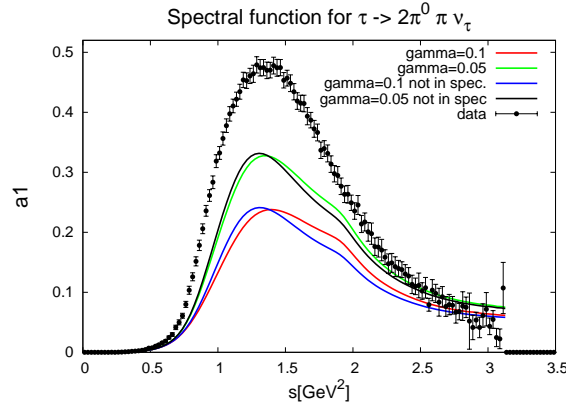
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Fig. 6. Simple model for the in-medium axial-vector spectral function. The ρ -meson propagator is modified by an additional constant in-medium width of 50 MeV (upper curves) or 100 MeV (lower curves), respectively. (The difference between the respective curves close by to each other is irrelevant for the present purpose.) Data taken from Ref. 4.

what happens to the dynamically generated a_1 . It must be stressed that this model should be regarded as a precursor to more serious calculations. E.g., a realistic in-medium width of the ρ -meson would not be independent of the momentum of the ρ -meson relative to the medium.¹⁴ In addition, one also expects a strong in-medium effect on the pion and not only on the ρ -meson.¹⁴ These aspects are not covered by the simple model studied here. The result is shown in Fig. 6. Obviously, the a_1 -meson also melts, if the ρ -meson melts. This does not prove that the melting scenario is the correct approach to chiral restoration, but at least we obtain a consistent picture. In a somewhat sloppy way one might say, that the problem of the missing partner of the ρ -meson on the single-particle level is solved by deconfinement.

References

1. V. Koch, *Int. J. Mod. Phys. E* **6**, 203 (1997).
2. M. E. Peskin and D. V. Schroeder, *An Introduction to Quantum Field Theory* (Perseus Books, Cambridge, Massachusetts, 1995).
3. Particle Data Group (W. M. Yao *et al.*), *J. Phys. G* **33**, 1 (2006).
4. ALEPH Collab. (S. Schael *et al.*), *Phys. Rept.* **421**, 191 (2005).
5. M. Wagner and S. Leupold, arXiv:0801.0814 [hep-ph].
6. L. M. Barkov *et al.*, *Nucl. Phys. B* **256**, 365 (1985); CMD-2 Collab. (R. R. Akhmetshin *et al.*), *Phys. Lett. B* **578**, 285 (2004).
7. J. Gasser and H. Leutwyler, *Nucl. Phys. B* **250**, 465 (1985).
8. M. F. M. Lutz and E. E. Kolomeitsev, *Nucl. Phys. A* **730**, 392 (2004).
9. T. Hyodo, D. Jido and A. Hosaka, arXiv:0803.2550 [nucl-th].
10. J. A. Oller and E. Oset, *Phys. Rev. D* **60**, 074023 (1999).
11. S. Leupold, in preparation.
12. S. Weinberg, *Phys. Rev. Lett.* **17**, 616 (1966); Y. Tomozawa, *Nuovo Cim.* **46A**, 707 (1966).
13. L. Roca, E. Oset and J. Singh, *Phys. Rev. D* **72**, 014002 (2005).

14. M. Post, S. Leupold and U. Mosel, *Nucl. Phys. A* **741**, 81 (2004).